Electricity & Magnetism

SEM - 2 (Hons)

Electromagnetic Induction: Faraday's Law. Lenz's Law.Self Inductance and Mutual Inductance.Reciprocity Theorem. Energy stored in a Magnetic Field. Introduction to Maxwell"s Equations. Charge Conservation and Displacement current. (6 Lectures)

Faraday's Law & Lenz's Law

A changing magnetic field induces an electric field.

Integral form of Faraday's law

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt},$$

Differential form of Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

-ve sign fixes the correct direction of emf: this is **Lenz's Law**

Nature abhors a change in flux.

Self induction

a changing current induces an emf in the source loop itself. The field and also the flux is proportional to current:

$$\Phi = LI.$$

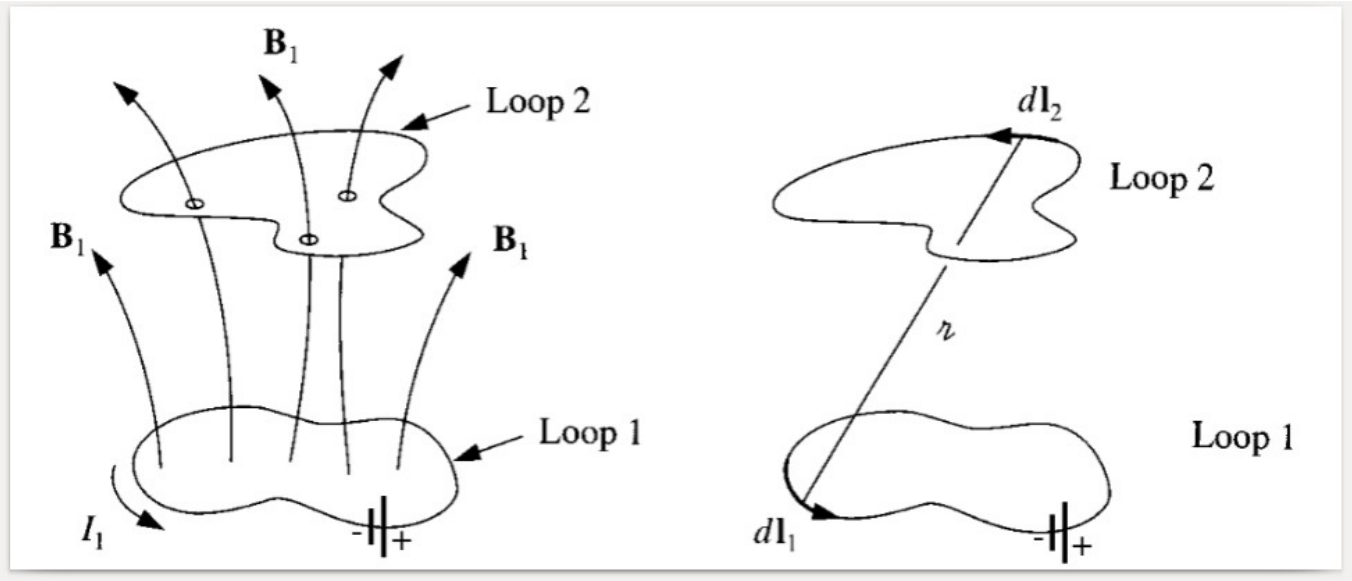
The constant of proportionality L is called the self-induction of the coil.

The induced emf is

$$\mathcal{E} = -L\frac{dI}{dt}.$$

Mutual Induction

Due to current flowing in loop 1, there is induced emf, thus current in loop 2



Look, there is battery in loop 1 and no battery in loop 2… The current in loop 2 is due to Induced emf.

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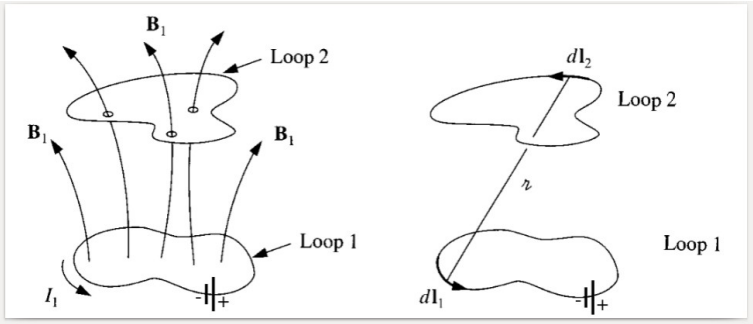

Mag field due to loop 1

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\mathbf{l}_1 \times \hat{\boldsymbol{r}}}{r^2},$$

Flux in loop 2

$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2.$$

$$\Phi_2 = M_{21} I_1,$$

$$\mathbf{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\mathbf{l}_1}{r},$$

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{r} \right) \cdot d\mathbf{l}_2$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r}.$$

$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2 = \int (\nabla \times \mathbf{A}_1) \cdot d\mathbf{a}_2 = \oint \mathbf{A}_1 \cdot d\mathbf{l}_2.$$

Induced emf in loop 2

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M \frac{dI_1}{dt}.$$

Reciprocity theorem

- You know this...

Energy stored in Magnetic field

$$W_{\text{elec}} = \frac{1}{2}\int (V\rho)\, d\tau = \frac{\epsilon_0}{2}\int E^2\, d\tau,$$

$$W_{\text{mag}} = \frac{1}{2}\int (\mathbf{A}\cdot\mathbf{J})\, d\tau = \frac{1}{2\mu_0}\int B^2\, d\tau.$$

In a coil with self induction ***L***

$$\frac{dW}{dt} = -\mathcal{E}I = LI\frac{dI}{dt}.$$

$$W = \frac{1}{2}LI^2.$$

Maxwell's Equations

Remember these four formulas for the rest of you life (If Corona virus spare you..)

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \text{(Gauss's law)},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \qquad \text{(no name)},$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law)},$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \text{(Ampère's law)}.$$

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Think of a capacitor

In 1888 (today is March, 2020) Maxwell found error in Eq. (iv)

Charge Conservation & Displacement current

Ask your teacher and clear your concept of

Free charge, Magnetic charge and Bound charge

New Ampere's law, with Maxwell's Correction term:

$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.}$$

Displacement current density

$$\mathbf{J}_d \equiv \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

total displacement current,

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{a}.$$

Some advise:

- Solve problems from competitive examinations.
- Solve the problems from the books 1) Mahajan Rangwala and 2) Griffiths
- Read your class 12 Physics book again..

Thank you for your questions..